An Analysis of Critical Film Take-up Velocity

presented to
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by
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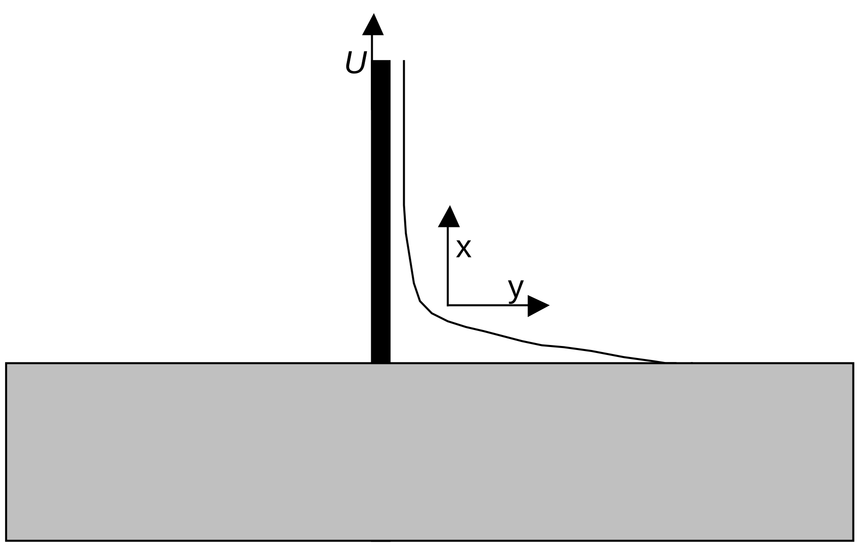
May 4, 2000

Motivation

The most effective geometry for gas separation membranes is that of a hollow fiber. This provides the maximum surface area to volume ratio and allows for a simple extrusion production process. During the extrusion (spinning), polymer is co-extruded with a bore fluid from a spinneret to form a nascent fiber, which is then quenched in water and taken up on a rotating drum. The drum provides a convenient way to store fibers while spinning, as well as providing the force to draw down the fibers from their initial extrusion dimensions to something more commercially relevant. The chief danger with the take-up drum is the drying of the fibers. If water-saturated fibers are allowed to air dry, the surface tension of the water will collapse the porous substructure, making the fibers ineffective for gas separations. In order to prevent this, the drum is set in a shallow pan of water, so that the surface of the drum will remain coated with water, keeping the fibers from drying. This project will attempt to analyze this drum and its water film and make an estimate of the minimum speed required to maintain a steady-state liquid film on the surface of the drum.

Model

As an approximation of the drum, a simple dip-coating system will be analyzed. An infinite plane will be drawn vertically from a fluid at velocity *U*. The analysis will attempt to determine if a there is a critical plate velocity, above which a steady-state coating solution exists. The planar approximation introduces very little error into the analysis, since the radius of the drum is very much greater than the thickness of the liquid film. The assumption of vertical withdrawal is a less valid assumption, the consequences of which will be addressed in the discussion section.



Solution
To determine the critical velocity, it will first be assumed that take-up does occur. If a valid solution to the fluid dynamics problem exists, then the assumption of take-up is valid.

For this system, the Navier-Stokes equations simplify to

(1) $$0 = -\frac{dP}{dx} + \mu \frac{d^2 u}{dx^2} + \rho\, g_x$$

where the pressure driving force is given by the capillary pressure

(2) $$-\Delta P = \frac{\gamma}{R} = \frac{\gamma \delta''}{(1+\delta'^2)^{3/2}} \approx \gamma \delta''$$

This results in the following governing differential equation, where δ is the thickness of the film, γ is surface tension, ρ is the density of the fluid, μ is the fluid viscosity, and u is the fluid velocity in the x-direction.

(3) $$0 = \gamma \frac{d^3 \delta}{dx^3} - \rho\, g + \mu \frac{d^2 u}{dy^2}$$

Upon applying the no-slip and zero stress boundary conditions at the plate and fluid-air interface, respectively, the equation can be solved for the velocity profile.

(4) $$u = -\frac{G}{\mu}\left(\frac{y^2}{2} - \delta y\right) + U$$

where $G = \gamma \frac{d^3 \delta}{dx^3} - \rho\, g$ and U is the velocity of the plate in the x-direction. Integrating to determine Q, the flow per unit width, gives

(5) $$Q = U\delta + \frac{G}{\mu}\frac{\delta^3}{3}$$

This equation applies everywhere along the film surface. However, at a far distance from the fluid bath, the film has thinned to a point at which the force of gravity is overwhelmed by the surface tension forces. Without the retarding force of gravity, the steady-state solution for Q simplifies to $Q = U\delta_f$, where δ_f is the far field, steady film thickness. This value can be found from the traditional treatments of dip coating, such as the one found in Probstein, where δ_f is given as

(6) $$\delta_f = (0.946)U\left(\frac{U\mu}{\gamma}\right)^{2/3}\sqrt{\frac{\gamma}{\rho g}}$$

Since (5) is valid for the flow rate anywhere, it can be applied to the intermediate regime between the meniscus and far-field regions. In this intermediate regime, the gravity drainage term is dominant over the surface tension. Additionally, to satisfy the conservation of volume, the gravity regime flow rate must equal the far-field flow rate.

(7) $$U\delta_f = Q = U\delta - \frac{\rho g}{\mu}\frac{\delta^3}{3}$$

Solving this equation for δ and ignoring the imaginary roots gives a gravity regime film thickness of

(8)
$$\delta = -\frac{1}{2}\frac{2^{2/3}\left\{\left[-\gamma^{17/9}\left[\mu U\rho^2 g^2\left(3(0.946)(\mu U)^{2/3}\sqrt{\tfrac{g}{\rho}}+\sqrt{\frac{\mu U\left(4\gamma^{1/3}+9(0.946)^2(\mu U)^{1/3}g^2\right)}{g\rho}}\right)\right]\right]^{2/3}+2^{2/3}\mu U\rho g\gamma^2\right]\right\}}{\rho g\gamma^{35/18}\left[\mu U\rho^2 g^2\left(3(0.946)(\mu U)^{2/3}\sqrt{\tfrac{g}{\rho}}+\sqrt{\frac{\mu U\left(4\gamma^{1/3}+9(0.946)^2(\mu U)^{1/3}g^2\right)}{g\rho}}\right)\right]^{1/3}}$$

Combining this expression with (5) gives an expression for flow rate that is dependant on the physical parameters of the system and the velocity of the plate. In order for take-up to occur, the flow rate at all times must be positive. Since the gravity drain regime is the most likely to produce a negative flow rate, satisfying the positive flow rate condition here will guarantee take-up. Substituting the appropriate equations, we find

(9) $$U \geq \frac{\rho g\delta^2}{3\mu}$$

Since δ is also a complicated function of U, (9) must be solved numerically to obtain the critical take-up velocity.

Conclusions and Discussion
The analysis presented herein shows that a critical take-up velocity can be determined for the model dip-coating system. However, because of the interconnected nature of the various parameters, an analytical solution is unwieldy at best. However, a numerical solution can be obtained by inputting typical values for the physical parameters. Using the density of water and a surface tension of 50 mJ/m^2, the following viscosity dependence was found.

Viscosity (cP)	1	100	10000
Plate Velocity (m/s)	1.36×10^{-1}	1.36×10^{-3}	1.36×10^{-5}

The critical plate velocity is inversely proportional to the viscosity, with more viscous materials requiring a lower plate velocity for take-up.

Upon applying this model problem to the real situation of the take-up drum, the two simplifications, the planar geometry and the zero plate-fluid bath angle must be dealt with. With regards to the geometry, the planar solution can be used without hesitation, due to the extremely large ratio of drum radius to film thickness.

The plate-bath angle, however, could have a large effect on the system in two ways. The first is the orientation of gravity. This could possibly be taken into account by simply determining the component of gravity acting in the plane of the take-up drum. Additional complications could also arise as gravity acts in opposition to the surface tension adhesion. This would undoubtedly affect the film thickness values. The other effect of a non-vertical plate would be on the meniscus at the bath surface. As the plate-bath angle is decreased, the meniscus radius of curvature would decrease, changing the surface tension-driven pressure gradient. A reworking of the steady-state film thickness would be required to take this into account.

Acknowledgements
I would like to thank Dr. Roger Bonnecaze for his patience and assistance with all phases of this project.

References
Probstein, Ronald F., Physicochemical Hydrodynamics, An Introduction, 2nd Ed., John Wiley and Sons, Inc., 1994, pp 318-323.