Prediction of Coating Thickness

John D. Wind

Surface Phenomena
ChE 385M

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Introduction

This project involves the modeling of the coating of metal plates with a viscous liquid by pulling the plate vertically through a pool of the liquid. The goal of the modeling is to predict the coating thickness as a function of the operational parameters in the system (plate velocity, temperature gradients, viscosity, density, and surface tension).

The liquid coating will cure or harden by the application of heat. To speed up the cure time, a proposed idea is to heat the plate by conducting heat through it. The concern is that because of the temperature gradients, the coating thickness may be affected because of a flow field derived from surface tension gradients. Thus, the momentum and energy balances are coupled. A schematic of the system is shown in Figure 1.

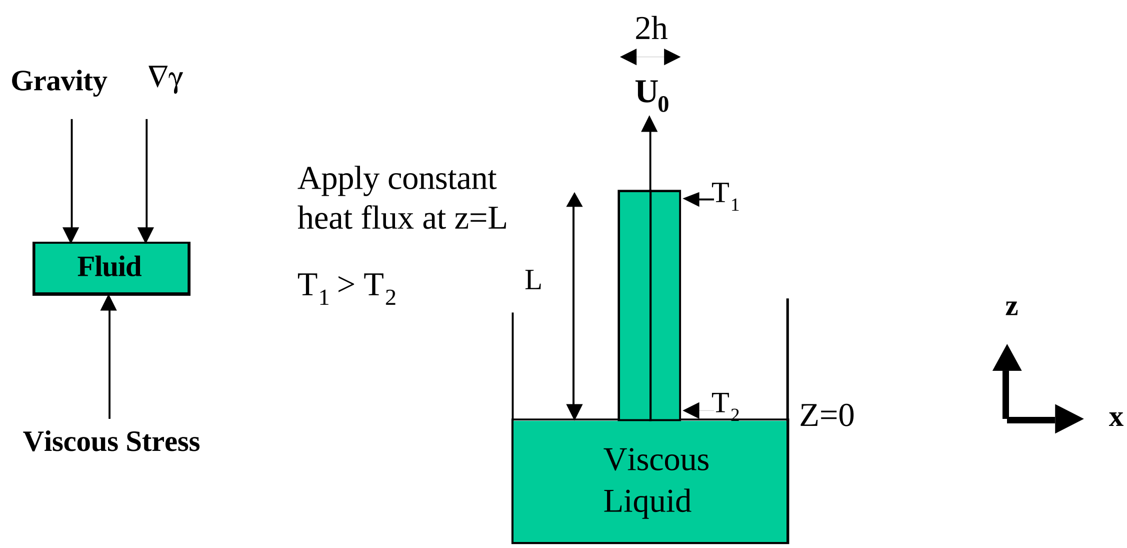

Figure 1: Schematic of Coating Apparatus

Assumptions

- Coating hardens at z > L (out of region for modeling)
- No temperature gradients for z > L
- No heat loss from edges of liquid coating to surrounding air
- Linear temperature profile from z = 0 to z = L
- Thin coating (dT/dx = 0)
- Newtonian fluid
- Laminar flow (no rippling)
- Fully developed flow at z = 0

Velocity Profile and Film Thickness

For this system, gravity and surface tension forces are balanced by viscous forces. The flow is unidirectional in the z-direction, so the Navier-Stokes equation reduces to

$$\mu \frac{\partial^2 v_z}{\partial x^2} = -\rho g_z$$

We wish to solve for the velocity profile in order to solve for the coating thickness *h*. The appropriate boundary conditions are:

at x = 0, $v_z = U_0$ (no slip)

at x = h, $\tau_{xz} = \frac{\partial \gamma}{\partial z} = \frac{\partial \gamma}{\partial T}\frac{\partial T}{\partial z} = \mu \frac{\partial v_z}{\partial x}$ (Marangoni stress)

We also know the following empirical relationship for the temperature dependence of surface tension:

$$\gamma = \gamma_0 \left(1 - \frac{T}{T_C}\right)^n$$

For many systems n = 11/9 but we will assume that n = 1 for simplicity.

This leads to the velocity profile

$$v_z = -\frac{\rho g}{2\mu} x^2 + C_1 x + C_2$$

The constants of integration are:

$C_1 = \frac{1}{\mu}\left(\frac{\partial \gamma}{\partial T}\frac{\partial T}{\partial z} + \rho \mathrm{gh}\right)$ and $C_2 = U_0$

$$v_z = \frac{1}{\mu}\left[-\frac{\rho g x^2}{2} + \left(-\frac{\gamma_0}{T_c}\frac{\partial T}{\partial z} + \rho \mathrm{gh}\right)x\right] + U_0 \qquad (1)$$

The dimensionless velocity profile for typical operating parameters (shown in Table 1 on page 5) is shown in Figure 1:

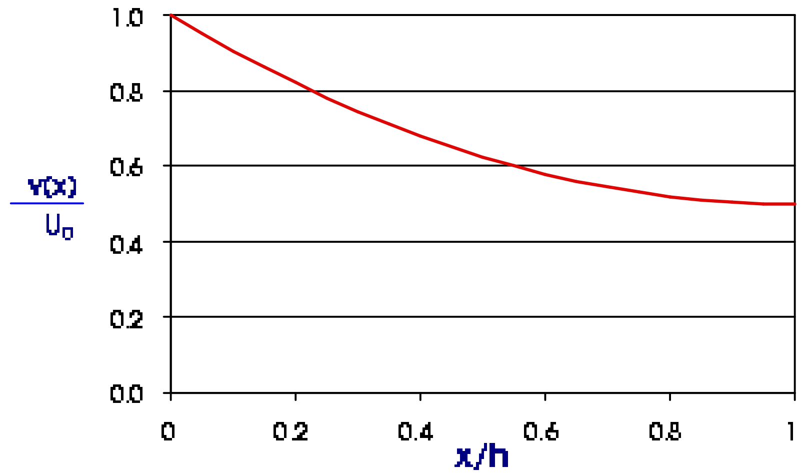

Figure 1: Velocity Profile for Typical Coating Operation

The thin film equation, shown in Equation 2, can be used to estimate the film thickness.

$$\frac{\partial h}{\partial t} + \nabla \bullet \int_0^h v_z dx = 0 \qquad (2)$$

For the steady-state value of *h*, we can set *dh/dt* equal to zero and solve for *h*.

$$\frac{d\left(\int_0^h v_z dx\right)}{dh} = 0 \qquad (3)$$

By inserting the velocity profile from Equation 1, the integral in Equation 3 is evaluated below:

$$\int_0^h \left\{ \frac{1}{\mu}\left[-\frac{\rho g x^2}{2} + \left(-\frac{\gamma_0}{T_c}\frac{\partial T}{\partial z} + \rho \mathrm{gh} \right) x \right] + U_0 \right\} dx = \frac{\rho g h^3}{3\mu} - \frac{1}{2\mu}\frac{\gamma_0}{T_c}\frac{\partial T}{\partial z} h^2 + U_0 h$$

and *h* is found by:

$$\text{or } h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a = \frac{\rho \mathrm{g}}{\mu} \text{ and } b = -\frac{\gamma_0}{\mu T_c}\frac{\partial T}{\partial z} \text{ and } c = U_0 \qquad (4)$$

The positive root for *h* is the only one that is physically realizable.

Stress Balance

We wish to find a dimensionless number that describes the effect of Marangoni forces on the film thickness. In the absence of surface tension, the film thickness can be calculated by Equation 5.

$$h_0 = \left(-\frac{\mu U_0}{\rho g} \right)^{1/2} \qquad (5)$$

This equation was derived in a similar fashion to that of Equation 4, except that the shear stress is zero at the liquid/air interface. This represents a balance between viscous and gravitational forces, as shown below:

$$\rho g h = \mu \frac{\partial v_z}{\partial x} = \mu \frac{U_0}{h}$$

When Marangoni stresses are considered, viscous forces are balanced by gravitational and surface tension forces:

$$\rho g h + \frac{\partial \gamma}{\partial z} = \mu \frac{U_0}{h}$$

As a result the dimensionless Wind number[1], is described in Equation 6.

$$Wi = \frac{\partial \gamma / \partial z}{\rho g h} \qquad (6)$$

Figure 2 shows how the film thickness is affected by the Marangoni stress where the thickness is normalized relative to the case where there are no Marangoni stresses.

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[1] Not well known in the literature

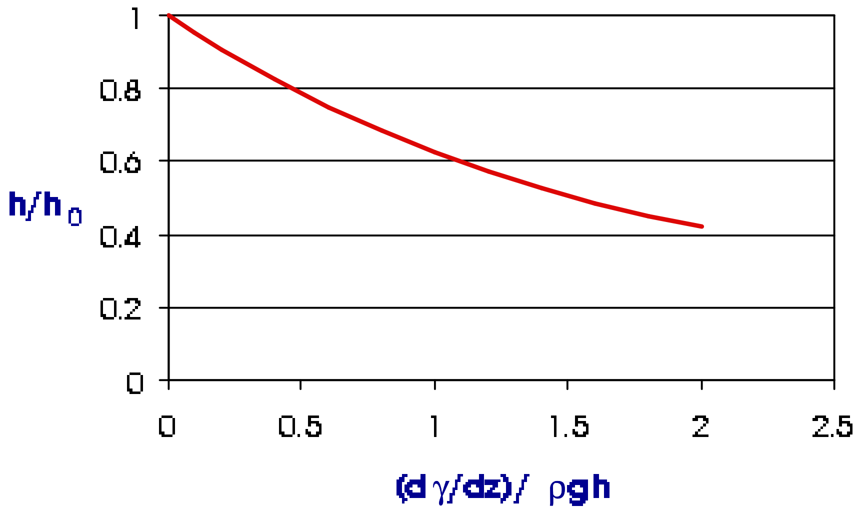

Figure 2: Effect of Marangoni Stress on Film Thickness

Practical Implications

Typical operating parameters for a coating operation are shown in Table 1:

Table 1: Operating Parameters for a Typical Coating Operation

Plate Velocity (U_0)	Viscosity (μ)	Density (ρ)	γ_0	T_c	dT/dz	Wi $(d\gamma/dz)/\rho gh$	Coating Thickness
m/s	cP	g/cm^3	dyne/cm	K	°C/m	dimensionless	mm
0.5	1200	1	180	500	5	2.3E-05	7.8

The value of the Wind number is so small that the Marangoni effect has a negligible effect on the coating thickness (see Figure 2). There would have to be a huge surface tension gradient to significantly affect the coating thickness.