Analysis of a Continuous Withdrawal Coating Process

Submitted to:
Dr. Roger Bonnecaze
ChE 385M – Surface Phenomena

Submitted by:
Stephen Sirard
The University of Texas at Austin
May 4, 2000

Analysis of a Continuous Withdrawal Coating Process

Introduction/Motivation

Coating films onto substrates is a very important process in many industrial applications. Coating films onto moving substrates finds applications in coating insulating layers onto wires, coatings on glass fibers, and in the preparation of photographic film or paper. In most applications it is desirable to predict the thickness of the film beforehand. In addition it is desirable to know how certain phenomena, such as temperature gradients, will affect the coating process. In this project, the continuous withdrawal of a substrate that entrains a liquid film is analyzed. In addition, surface tension gradients that could arise due to temperature gradients on the film surface are briefly addressed. The thickness of a film adhering onto a moving support is derived as a function of the pure fluid properties.

Problem Description

A diagram of a continuous withdrawal coating process is given in Figure 1:

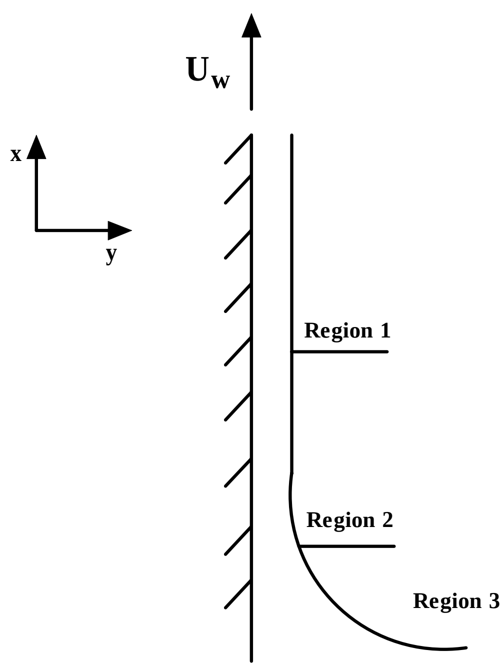

Figure 1: Continuous Withdrawal Coating Process

In this problem, the velocity profiles of a Newtonian fluid in regions 1 and 2 were solved assuming there was a temperature gradient in the film (Marangoni stress). In addition, the flow rate per width, Q, was derived for these two regions. Ultimately it is desirable to obtain the eventual thickness of the film in terms of the pure fluid properties. In solving the model for this thickness, the Marangoni condition was dropped in order to simplify the problem.

Model Formulation

The coating process can be separated into three regions. Each region contains certain assumptions that describe the flow. In general, it assumed that the flow is laminar, steady-state, and the fluid is Newtonian. More specifically, in region 1 it is assumed that the thickness of the film is constant so $dh/dx = 0$. Therefore the equations of motion simplify to the following equation

$$\mu \frac{d^2 u_x}{dy^2} = -\rho g_x \tag{1}$$

The boundary conditions in this region are as follows,

$$y = 0 \rightarrow u_x = U_w \tag{2}$$

$$y = h \rightarrow \mu \frac{du_x}{dy} = \frac{d\gamma}{dx} \tag{3}$$

where U_w is the velocity of the plate. In region 2, it is assumed that the flow is one-dimensional, $dh/dx << 1$, and that there is a negligible effect of flow on interfacial pressure change. With these assumptions, the flow model simplifies to

$$\gamma \frac{d^3 h}{dx^3} + \frac{d\gamma}{dx}\frac{d^2 h}{dx^2} + \mu \frac{d^2 u_x}{dy^2} + \rho g_x = 0 \tag{4}$$

where the boundary conditions are the same as those in Equations 2 and 3. Regions 1 and 2 can be matched by setting their flow rates per width equal. In other words,

$$Q_1 = \int_0^h u_x dy = Q_2 = const. \tag{5}$$

In region 3, it is assumed that the meniscus is static. Therefore, the shape of a surface with a static meniscus is given in Equation 6

$$\frac{\frac{d^2h}{dx^2}}{[1+\left(\frac{dh}{dx}\right)^2]^{3/2}} = \frac{\rho g x}{\gamma} \tag{6}$$

In order to obtain an equation for the thickness, regions 2 and 3 have to be matched. This matching will occur in a region where the curvature predicted from region 2 is the same as the curvature predicted by region 3. The principal of equal curvatures in the matching region is necessary for an unbroken surface. The matching curvatures are expected to occur in a region where *h(x)/ho* is approaching one for the static meniscus region, and where *h(x)/ho* is approaching infinity for region 2 (where *ho* is the final film thickness). Therefore the proper matching condition is

$$\left(\frac{d^2h}{dx^2}\right)_{h\to\infty}^{\text{Region2}} = \left(\frac{d^2h}{dx^2}\right)_{h\to ho}^{\text{Region3}} \tag{7}$$

Solution/Results

Solving Equation 1 for the velocity in Region 1 with a Marangoni stress condition, led to the following profile

$$u_x = \frac{-\rho g_x y^2}{2\mu} + \frac{y}{\mu}\left(\frac{d\gamma}{dx} + \rho g_x h\right) + U_w \tag{8}$$

From this velocity profile, the flow rate was calculated in Equation 9

$$Q_1 = \frac{-\rho g_x h^3}{6\mu} + \frac{h^2}{2\mu}\left(\frac{d\gamma}{dx} + \rho g_x h\right) + U_w h \tag{9}$$

The velocity profile and flow rate were also solved for the dynamic meniscus region, i.e. region 2, as shown in Equations 10 and 11

$$u_x = \frac{y^2}{2\mu}\left(-pg_x - \frac{\gamma d^3h}{dx^3} - \frac{d\gamma}{dx}\frac{d^2h}{dx^2}\right) + \frac{y}{\mu}\left(\frac{d\gamma}{dx} + h\left(pg_x + \gamma\frac{d^3h}{dx^3} + \frac{d\gamma}{dx}\frac{d^2h}{dx^2}\right)\right) + U_w \tag{10}$$

$$Q_2 = U_w h + \frac{h^2}{2\mu}\frac{d\gamma}{dx} + \frac{h^3}{3\mu}\left(\rho g_x + \gamma\frac{d^3h}{dx^3} + \frac{d\gamma}{dx}\frac{d^2h}{dx^2}\right) \tag{11}$$

At this point, the Marangoni effect was dropped for simplicity in order to determine the thickness of the film in terms of pure fluid properties. By setting $Q_1 = Q_2$, and setting h in Q_1 to the final thickness value, a differential equation results. After nondimensionalizing the equations and assuming very small values of the capillary number, Equation 12 results

$$\frac{d^3L}{d\lambda^3} - \frac{(1-L)}{L^3} = 0 \qquad (12)$$

where L and λ are defined as follows

$$L = \frac{h(x)}{h_\infty} \qquad (13)$$

$$\lambda = \left(3\frac{\mu U_w}{\gamma}\right)^{1/3} \frac{x}{h_\infty} \qquad (14)$$

Regions 2 and 3 have to be matched as described previously. From equation 12, and the condition specified in Equation 7, it is seen that the curvature for region 2 approaches a constant as L approaches infinity. For region 3, Equation 6 can be integrated with the condition that the thickness approaches infinity at the water line. This allows the curvature to be solved as h approaches the final thickness (or as L approaches 1). After setting the curvatures equal, Equation 15 results

$$h_\infty = \alpha(2\rho g)^{-1/2}(3\mu U_w)^{2/3}(\gamma)^{-1/6} \qquad (15)$$

where, α is an unknown constant that could be fit to experimental data, or perhaps determined from a numerical solution of Equation 12.

Equation 15 shows that the final film thickness depends on the plate withdrawal velocity as well as the density, viscosity, and surface tension of the coating solution. In addition, it can be concluded that the surface tension effects on the final film thickness are small. Therefore, applying “surface active” agents to the solvent probably would not affect the final film thickness significantly. Plots of the final film thickness as a function of plate withdrawal velocity are shown in Figures 2 and 3 for different solvents (water, heptane, and liquid carbon dioxide.) and different values of α.

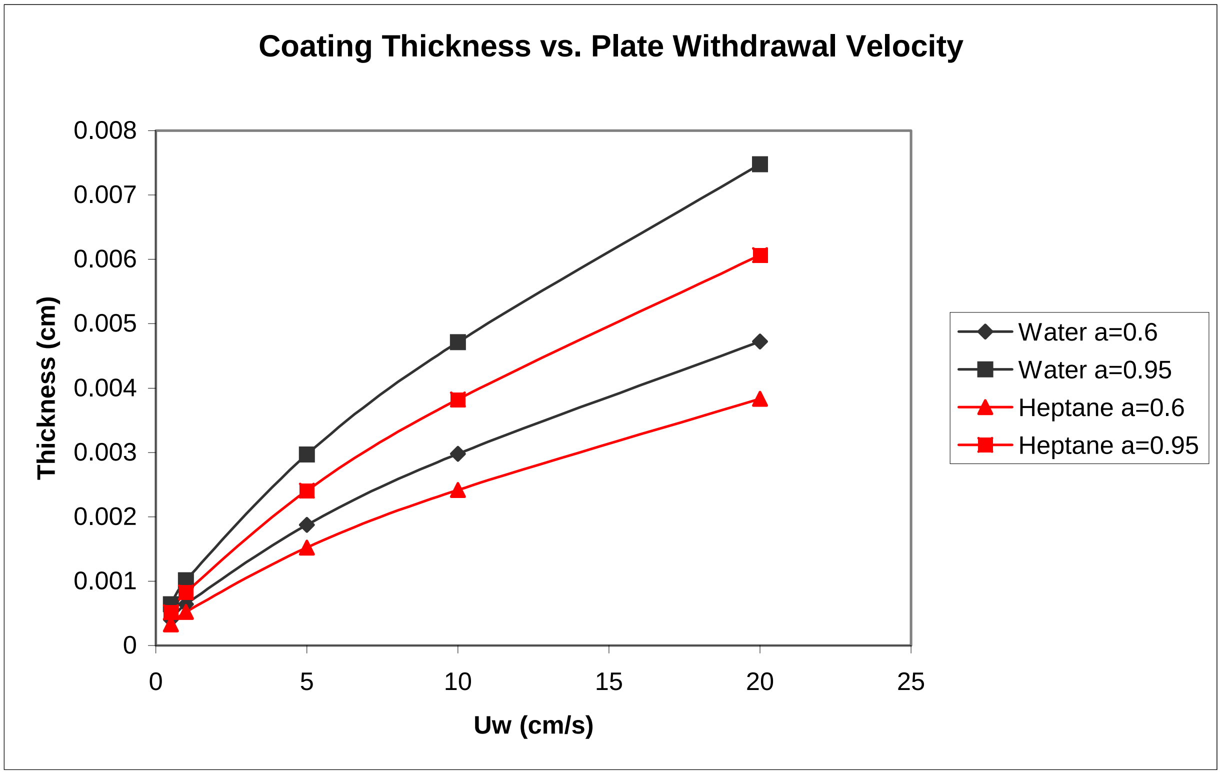

Figure 2 Effect of α, plate withdrawal velocity, and solvent on final film thickness.

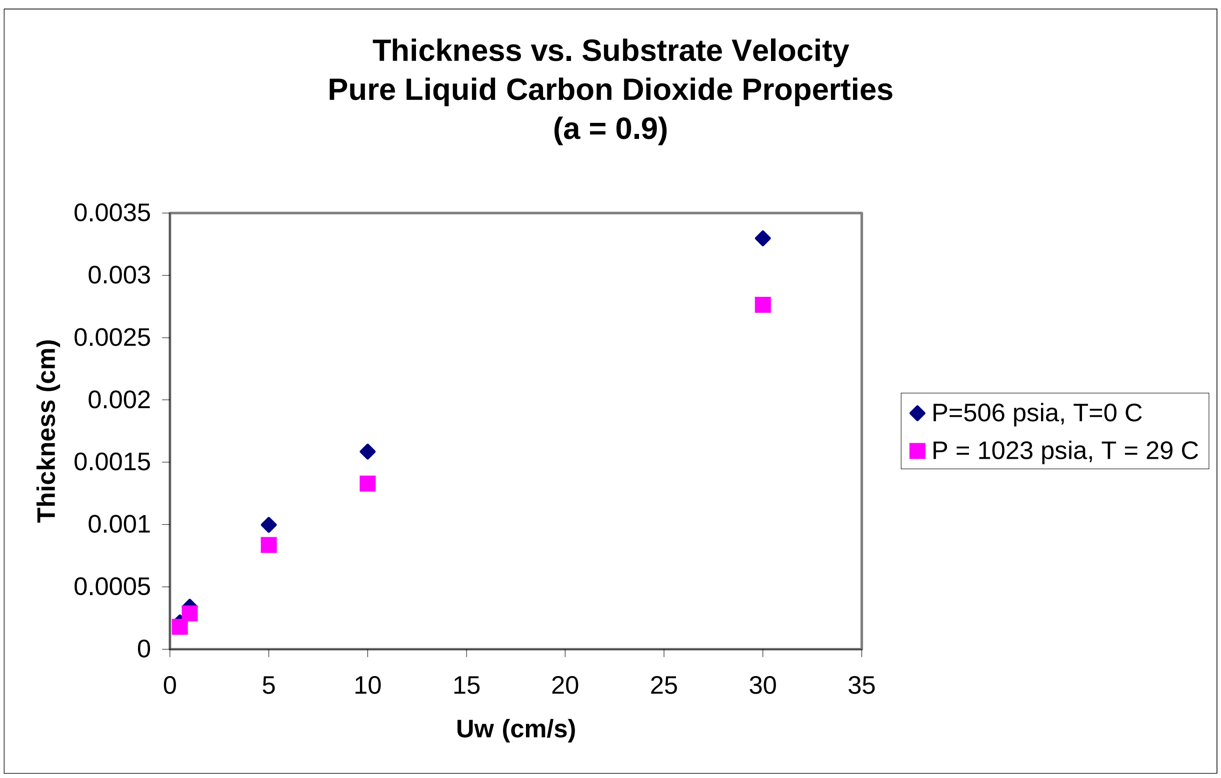

Figure 3 Effect of "tuning" CO_2 on the final film thickness.

References

1)Bird, R. B.; Stewart, W. E.; Lightfoot, E. N. *Transport Phenomena*; John Wiley & Sons: New York, 1960.

2)Gutfinger, C.; Tallmadge, J. A. *A.I.Ch.E. Journal* **1965**, *11*, 403-413.

3)Levich, V. G. *Physicochemical Hydrodynamics*; Prentice-Hall: Englewood Cliffs, 1962.

4)Novick, B. J. C., R.G.; Desimone, J.M. *Proceedings of the 5th International Symposium on Supercritical Fluids* **2000**.